Questions in Action Models

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Thom van Gessel

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Institute for Logic, Language and Computation

Questions often guide information updates:

- Information exchange through communication: asking *questions* and providing *answers*;
- Scientific inquiry: performing experiments and obtaining results, formulating a research agenda.

Introduction

We can describe these processes more accurately if we add *questions* to DEL.

- DELQ (van Benthem & Minică 2012)
- IDEL (Ciardelli & Roelofsen 2015)

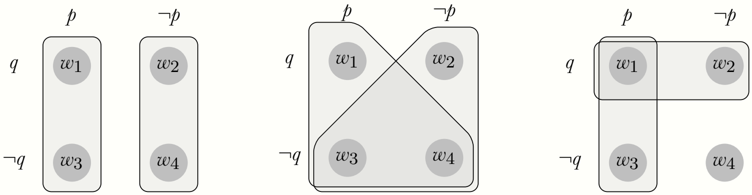
Statements convey *information*, while questions raise *issues*.

DELQ & IDEL

	DELQ	IDEL
Issues	Partition	Inquisitive
Formulas	Statements	Statements & questions

Table 1: Comparison of DELQ and IDEL

Partition vs inquisitive notion of issues



(a) Polar question

(b) If John comes to the party (q), does Mary come (p)?

(c) Does John speak English (p), or French (q)?

Figure 1: Not all issues are partition issues

DELQ & IDEL

	DELQ	IDEL
Issues	Partition	Inquisitive
Formulas	Statements	Statements & questions

Table 1: Comparison of DELQ and IDEL

Embedded questions

"Anna wonders whether Peter is coming to the party"

$$Wa?p$$

"If Peter comes to the party, does Quinn come?"

$$p \to ?q$$

Problem

IDEL only encodes *public* announcements.

Sometimes, not everyone is aware of what is being stated or asked.

Related logics

	No questions	Questions based on partition semantics	Questions based on inquisitive semantics
Static	EL	ELQ	IEL
Dynamic with public announcements	PAL	DELQ	IDEL
Dynamic with public and private announcements	AML	ELQm	

Table 2: Overview of standard, partition-based and inquisitive epistemic logics

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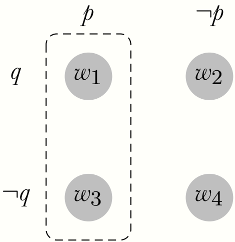
Inquisitive Epistemic Logic

Formulas are evaluated in *information states*:



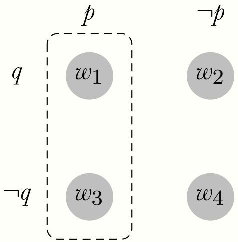
$$\models p$$

$$s \models \varphi \mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}} \psi$$

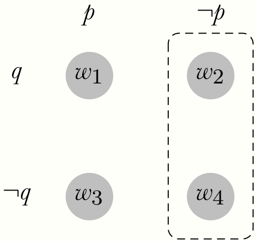
$$\Longleftrightarrow$$

$$s \models \varphi \text{ or } s \models \psi$$

Inquisitive Epistemic Logic



$$\models p \mathbin{\backslash\!\!/\!\!\!\!\backslash\!\!/} \neg p$$



$$\models p \vee\!\!\!\vee \neg p$$

$$?\varphi := \varphi \mathbin{\rotatebox[origin=c]{-90}{$\geqslant$}} \neg\varphi$$

Inquisitive Epistemic Logic

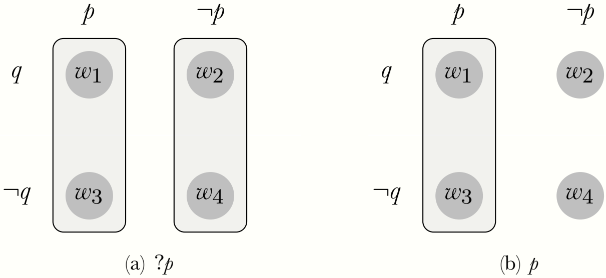


Figure 2: Question vs statement

Inquisitive Epistemic Model (Ciardelli & Roelofsen, 2015)

An inquisitive epistemic model is a triple $M = \langle W, \{\Sigma_a \mid a \in \mathcal{A}\}, V\rangle$ where:

- W is the domain of worlds;
- $\mathcal{A}$ is the domain of agents;
- For $w \in W$, $\Sigma_a(w)$ is a downward closed set of information states;
- V is a valuation function.

For each world w, a's information state $\sigma_a(w) := \bigcup \Sigma_a(w)$.

Example

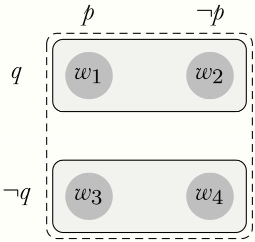


Figure 3: A state map in an inquisitive epistemic model

Inquisitive Epistemic Logic

Syntax

$\varphi ::= p \mid \bot \mid \varphi \wedge \varphi \mid \varphi \to \varphi \mid \varphi \mathbin{\mathbb{V}} \varphi \mid K_a\varphi \mid E_a\varphi$

Semantics

$$M, w \models K_a\varphi \quad \Longleftrightarrow \quad M, \sigma_a(w) \models \varphi$$

$$M, w \models E_a\varphi \quad \Longleftrightarrow \quad \text{for all } t \in \Sigma_a(w) : M, t \models \varphi$$

- $K_a\varphi$: the information state of *a* supports φ (*a* knows φ)
- $E_a\varphi$: all information states *a* desires to be in support φ (*a* entertains φ)

Dynamics

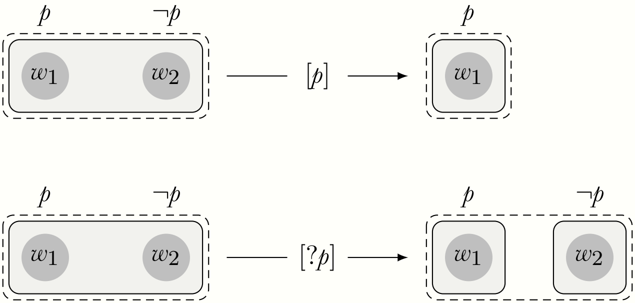


Figure 4: Public utterance in IDEL

Action Models with Questions

Action Models with Questions

In standard action models (Baltag et al. 1998), the *precondition* of an epistemic action determines what the action conveys (its *content*).

In a setting with questions, these two notions come apart.

Action Models with Questions

The *content* is a formula that expresses what the action conveys.

The *precondition* is a formula that expresses what has to be true for the action to be truthfully executed.

Example: if the content of an action is $p \mathbin{\vee\!\!\!\vee} q$, its precondition is $p \vee q$.

Action Models with Questions

Definition

An AMLQ action model is a triple $\mathsf{M} = \langle \mathsf{S}, \{\sim_a \mid a \in \mathcal{A}\}, \mathsf{cont}\rangle$, where:

- S is a finite domain of action points;
- For each $a \in \mathcal{A}$, $\sim_a$ is an equivalence relation on S;
- $\mathsf{cont} : \mathsf{S} \to \mathcal{L}^{\mathsf{IEL}}$ is a function that assigns a content $\mathsf{cont}(\mathsf{x}) \in \mathcal{L}^{\mathsf{IEL}}$ to each action point $\mathsf{x} \in \mathsf{S}$.

Given an action x, its precondition $\mathsf{pre}(\mathsf{x})$ is a formula expressing the informative content of $\mathsf{cont}(\mathsf{x})$.

Action Models with Questions

Update procedure (1/2)

$M' = (M \otimes \mathsf{M})$ is the product update of M and M, defined as follows.

$M' = \langle W', \{\Sigma'_a \mid a \in \mathcal{A}\}, V' \rangle$, where:

- $W' = \{\langle w, \mathsf{x} \rangle \mid w \in W, \mathsf{x} \in \mathsf{S}$ and $M, w \models \mathsf{pre}(\mathsf{x})\}$
- $\langle w, \mathsf{x} \rangle \in V'(p)$ iff $w \in V(p)$
- $s \in \Sigma'_a(\langle w, \mathsf{x} \rangle)$ iff ...

Action Models with Questions

Some assumptions:

- No forgetting
- Curious agents:
  - They will entertain all questions they think are asked;
  - They either know or *want* to know which action happens.

Action Models with Questions

Projection operator

$$\pi_1(s) := \{w \mid \langle w, \mathsf{x} \rangle \in s \text{ for some } \mathsf{x}\}$$

$$\pi_2(s) := \{\mathsf{x} \mid \langle w, \mathsf{x} \rangle \in s \text{ for some } w\}$$

Update procedure (2/2)

$s \in \Sigma'_a(\langle w, \mathsf{x} \rangle)$ iff ...

(i)

(ii)

(iii)

(iv)

Action Models with Questions

Projection operator

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Update procedure (2/2)

$s \in \Sigma'_a(\langle w, \mathsf{x}\rangle)$ iff ...

(i) $\pi_1(s) \in \Sigma_a(w)$ — no forgetting, also for issues

(ii)

(iii)

(iv)

Action Models with Questions

Projection operator

$$\pi_1(s) := \{w \mid \langle w, \mathsf{x}\rangle \in s \text{ for some } \mathsf{x}\}$$

$$\pi_2(s) := \{\mathsf{x} \mid \langle w, \mathsf{x}\rangle \in s \text{ for some } w\}$$

Update procedure (2/2)

$s \in \Sigma'_a(\langle w, \mathsf{x}\rangle)$ iff ...

(i) $\pi_1(s) \in \Sigma_a(w)$

(ii) $\forall \mathsf{y} \in \pi_2(s) : \mathsf{x} \sim_a \mathsf{y}$ — no forgetting, with respect to actions

(iii)

(iv)

Action Models with Questions

Projection operator

$$\pi_1(s) := \{w \mid \langle w, \mathsf{x}\rangle \in s \text{ for some } \mathsf{x}\}$$

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(ii) $\forall \mathsf{y} \in \pi_2(s) : \mathsf{x} \sim_a \mathsf{y}$

(iii) There is at most one $\mathsf{y} \in \pi_2(s)$ states specify an action (curiosity)

(iv)

Action Models with Questions

Projection operator

$$\pi_1(s) := \{w \mid \langle w, \mathsf{x} \rangle \in s \text{ for some } \mathsf{x}\}$$

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Update procedure (2/2)

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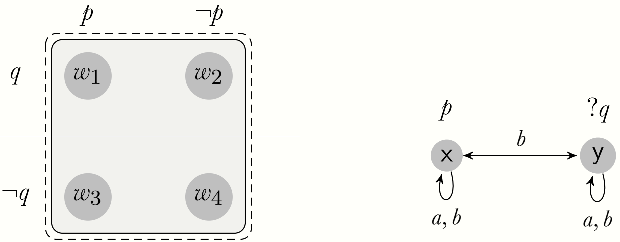
(i) $\pi_1(s) \in \Sigma_a(w)$

(ii) $\forall \mathsf{y} \in \pi_2(s) : \mathsf{x} \sim_a \mathsf{y}$

(iii) There is at most one $\mathsf{y} \in \pi_2(s)$

(iv) $\forall \mathsf{y} \in \pi_2(s) : M, \pi_1(s) \models \mathsf{cont}(\mathsf{y})$ action content can raise issues (curiosity)

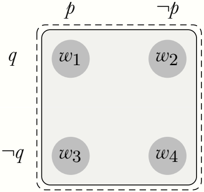
Example



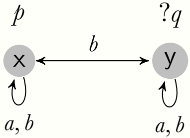
(a) State map of *a* and *b* before action (b) Action model

Figure 5: Original model and action model

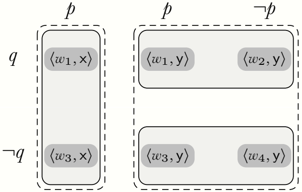
Example



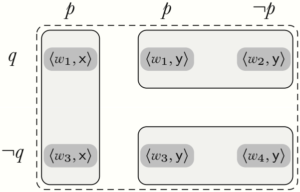
(a) State map of *a* and *b* before action



(b) Action model



(c) State map of *a* after action



(d) State map of *b* after action

Figure 6: Original model, action model and product update

Logical language

Dynamic modalities

Single actions or sets of actions

$[\mathsf{x}]\varphi$: 'after x, our information supports φ'
$[\mathsf{s}]\varphi$: 'incorporating the information that some action in s has taken place, our information supports φ'

Let s be a state in M and let t be a state in M. Let $M' = M \otimes \mathsf{M}$.

Updated state

$$s[\mathsf{t}] = \{\langle w, \mathsf{x}\rangle \in W' \mid w \in s \text{ and } \mathsf{x} \in \mathsf{t}\}$$

Support condition

$$M, s \models [\mathsf{t}]\varphi \iff M', s[\mathsf{t}] \models \varphi$$

Dynamic modalities

A set modality is interpreted as partial information about the action taking place. Therefore we have:

Proposition

$$[\{\mathsf{x}, \mathsf{y}\}]\varphi \not\equiv [\mathsf{x}]\varphi \wedge [\mathsf{y}]\varphi$$

Since φ may be a question that is supported both after x and after y, but not in the same way.

Proposition

If α is a statement, then

$$[\{\mathsf{x}, \mathsf{y}\}]\alpha \equiv [\mathsf{x}]\alpha \wedge [\mathsf{y}]\alpha$$

Axiomatization

Axiomatization

By means of reduction to IEL:

$$[\mathsf{s}]\alpha \equiv \bigwedge_{\mathsf{x}\in\mathsf{s}} [\mathsf{x}]\alpha$$

$$[\mathsf{x}]p \equiv \mathsf{pre}(\mathsf{x}) \to p$$

$$[\mathsf{x}]\bot \equiv \neg\mathsf{pre}(\mathsf{x})$$

$$[\mathsf{s}](\varphi \wedge \psi) \equiv [\mathsf{s}]\varphi \wedge [\mathsf{s}]\psi$$

$$[\mathsf{s}](\varphi \mathbin{\vee\!\!\!\vee} \psi) \equiv [\mathsf{s}]\varphi \mathbin{\vee\!\!\!\vee} [\mathsf{s}]\psi$$

$$[\mathsf{x}](\varphi \to \psi) \equiv [\mathsf{x}]\varphi \to [\mathsf{x}]\psi$$

$$[\mathsf{x}]K_a\varphi \equiv \mathsf{pre}(\mathsf{x}) \to K_a[[\![\mathsf{x}]\!]_{\sim_a}]\varphi$$

$$[\mathsf{x}]E_a\varphi \equiv \mathsf{pre}(\mathsf{x}) \to \bigwedge_{\mathsf{y}\sim_a\mathsf{x}} E_a(\mathsf{cont}(\mathsf{y}) \to [\mathsf{y}]\varphi)$$

Where α ranges over statements, and $[\![\mathsf{x}]\!]_{\sim_a}$ is a's equivalence class of x.

Conclusion

Conclusion

We can use Action Models with Questions to describe private exchanges of information *and questions*, and compute their effects.

The logic AMLQ contains AML and IEL.

It is a conservative extension of both.

It generalizes IDEL to private utterances in a natural way.

Suggestions for further work

- Adding *issues* to action models
- Doxastic logic, belief revision
- Common issues
- Protocols, questioning strategies

References

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