A Priori and Necessary Questions

Semantics & Philosophy in Europe

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A priori or necessary

We distinguish two kinds of true statements: *a priori* truths and *necessary* truths (Kripke, 1980).

- A statement is a priori true iff its truth can be established before experience.
- A statement is necessarily true iff it could not have been false.

A priori

(1) I am here now.

(2) I am in Poland now.

The truth of (1) can be established without checking where I am.

Necessary

(2) I am in Poland now.

(3) I am Thom.

While (2) could have been false, the same is not the case for (3).

A priori and necessary questions

Something similar seems to be going on when it comes to questions:

(4) Am I here now?

(5) Am I in Poland now?

(4) can be resolved without knowing what the world is like, but whether I am here now is contingent.

A priori and necessary questions

(6) Who am I?

(7) Where am I?

It is contingent where I am, but not who I am. Still, someone can fail to know who I am.

A priori and necessary questions

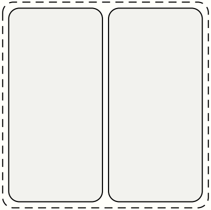
We observe that there is a sense in which (4) is a priori and contingent, while (6) is a posteriori and necessary.

(4) Am I here now?

(6) Who am I?

Can this be captured by existing frameworks for question semantics?

question meaning = set of resolving information states

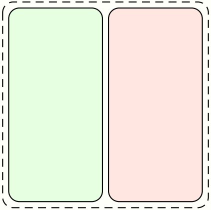


An information state *s* is informative enough to resolve 'Is John at home?' iff either

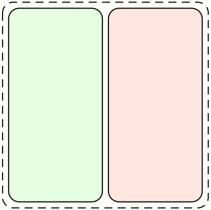
- in all worlds in *s*, John is at home or
- in all worlds in *s*, John is not at home

(4) Am I here now?

If we suppose that worlds specify a time, place and agent of utterance:



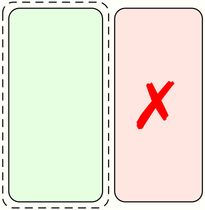
Strategies available in standard inquisitive semantics:



Include worlds where
the speaker is not at
the place of utterance
at the time of utterance

$\Downarrow$

No account of why
the question is a priori



Exclude these worlds

$\Downarrow$

No account of why
the question is contingent

It seems that standard inquisitive semantics is not rich enough to distinguish apriority from necessity.

In truth conditional semantics, this distinction can be made by going *two-dimensional*.

Goal: combine inquisitive semantics with two-dimensional semantics and define uniform notions of apriority and necessity that apply to both questions and statements.

Plan

1. Two-dimensional semantics
2. Lift to inquisitive semantics
3. Discussion of other frameworks (time permitting)
4. Conclusion

Two-dimensional semantics

Content & character

In standard possible worlds semantics, meaning is equated with truth conditions.

Formally: statements express a proposition, which is the set of worlds where it is true.

In two-dimensional semantics (e.g. Kaplan 1989), we distinguish character and content:

- The *content* of a statement is the set of worlds in which it is true.
- The *character* of a statement is a function from contexts to contents.

Example

(8) We are now in Poland.

In the present context, the content of (8) is the proposition 'that on September 22, the participants of SPE are in Poland'.

This proposition is true in the actual world, but false in others.

w_1	w_2	w_3	w_4	...
T	F	F	T	

In other contexts, e.g. next week in the Netherlands, the sentence would express a different proposition, true and false in different worlds.

w_1	w_2	w_3	w_4	...
F	T	T	F	

Sentences in general translate into a character:

	w_1	w_2	w_3	w_4	...
c_1	T	F	F	T	
c_2	F	T	T	F	
...					

Contexts

What are contexts?

- For Kaplan (1989), a context c is a tuple $\langle a_c, t_c, p_c, w_c \rangle$ (agent, time, position and world).
- For others (e.g. Stalnaker 1978) contexts are (centered) worlds

I simplify for the purposes of this talk:

- Contexts and worlds are the same kind of object
- Every world/context w has an agent a_w and position p_w such that a_w is at p_w in w (in Kaplan's terms: only *proper* contexts)
- We don't consider time

A model M is a structure $\langle W, A, P, I \rangle$ consisting of worlds/contexts, individuals, positions, and an interpretation function.

$[\alpha]_{cfw}$ = denotation of α with respect to context c, assignment function f and world w.

- $[\mathrm{I}]_{cfw} = a_c$
- $[\mathrm{here}]_{cfw} = p_c$
- $[\mathrm{Located}]_{cfw} = \{\langle x, y \rangle \mid x \text{ is at } y \text{ in } w\}$

- If a is any other constant, $[a]_{cfw} = I(a)(w)$
- If x is a variable, $[x]_{cfw} = f(x)$

Necessity and apriority

- α expresses a necessary proposition in c iff all worlds make this proposition true.

$$c, w \models_f \Box\alpha \iff \text{for all } w' : c, w' \models_f \alpha$$

- α is a priori iff for all contexts c', the proposition expressed by α is true in c' itself.

$$c, w \models_f \blacksquare\alpha \iff \text{for all } c'\text{: } c', c' \models_f \alpha$$

Examples

Let @ be the actual world/context. Then:

- $@, @ \models \blacksquare \mathit{Located}(I, \mathit{here})$
  For all c: $c, c \models \mathit{Located}(I, \mathit{here})$
- $@, @ \models \Box(I = \mathit{Thom})$
  For all w: $@, w \models I = \mathit{Thom}$

Bonus: Actuality operator

Two-dimensional semantics also allows us to define an Actuality operator:

$$c, w \models_f A\alpha \iff c, c \models_f \alpha$$

This allows us to express sentences like (9):

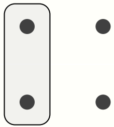
(9) The (actually) rich could have all been poor
$\Diamond\forall x(ARx \rightarrow Px)$

Two-dimensional inquisitive semantics

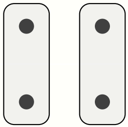
Inquisitive semantics

- Sentences are evaluated not relative to worlds (truth conditions), but relative to *information states* (resolution conditions)
- Propositions are downward closed sets of information states, either with more than one maximal element (questions) or with one (statements)
- Questions are introduced by inquisitive disjunction or inquisitive existential quantification:

$$s \models \varphi \mathbin{\vee\!\!\!\vee} \psi \iff s \models \varphi \text{ or } s \models \psi$$



p



$p \mathbin{\vee\!\!\!\vee} \neg p$

Combining 2D with inquisitive semantics

Two-dimensional semantics:

- statement $\rightsquigarrow$ character
- character: contexts $\to$ contents (truth conditions)

In inquisitive semantics, truth conditions are replaced by *resolution conditions*.
First suggestion:

- sentence $\rightsquigarrow$ character
- character: contexts $\to$ contents (resolution conditions)

Example

(10) Am I in Poland?

Intuitively, the content of (10) (in the present context) is ‘Is Thom in Poland?’, which is resolved in information states that specify either that Thom is in Poland or that he isn’t.

Thus, the character is a function which takes a context c and returns the set of information states that resolve the question whether a_c is in Poland.

Semantics

We write $c, s \models_f \varphi$ to indicate that information state s supports what φ expresses in c, under assignment function f.

- $c, s \models_f Qa_1...a_n \iff$ for all $w \in s$: $\langle [a_1]_{cfw}, ..., [a_n]_{cfw} \rangle \in [Q]_{cfw}$
- $c, s \models_f a = b \iff$ for all $w \in s : [a]_{cfw} = [b]_{cfw}$
- $c, s \models_f \bot \iff s = \emptyset$
- $c, s \models_f \varphi \wedge \psi \iff c, s \models_f \varphi$ and $c, s \models_f \psi$

⋆ $c, s \models_f \varphi \bigvee \psi \iff c, s \models_f \varphi$ or $c, s \models_f \psi$

- $c, s \models_f \varphi \rightarrow \psi \iff$ for all $t \subseteq s : c, t \models_f \varphi$ implies $c, t \models_f \psi$

⋆ $c, s \models_f \exists x \varphi \iff$ there is some $a \in A$ such that $c, s \models_{f_a^x} \varphi$

⋆ $c, s \models_f A\varphi \iff c, \{c\} \models_f \varphi$

$\neg\varphi := \varphi \rightarrow \bot$ $\qquad\qquad$ $\varphi \vee \psi := \neg(\neg\varphi \wedge \neg\psi)$

Necessity and apriority

The standard definitions of necessity and apriority are based on truth in a world: this would make questions like 'is p the case or not?' a priori and necessary.

- φ expresses a necessary proposition in c iff the maximal information state supports it.

$$c, s \models_f \Box\varphi \iff c, W \models_f \varphi$$

- φ is a priori iff for each context c', the maximal information state that is proper for c' supports φ.

$$c, s \models_f \blacksquare\varphi \iff \text{for all } c' \text{: } c', s_{c'} \models_f \varphi$$

Where s_c is the set of worlds that are 'proper' for c (for our purposes, the ones where a_c is at p_c).

Problem

(11) Where am I?
$\exists x Located(I, x)$

This approach predicts that (11) is a priori: for any context c, the location of the speaker (p_c) will be fixed across all worlds in s_c.

This is wrong: the question cannot be resolved without experience, we need to know who the speaker is and where they are.

Problem

The resolution conditions of a question like (11) cannot be obtained by fixing a context, because the question is about *what the context is like*.

- character: contexts $\rightarrow$ contents (resolution conditions)

Resolution conditions should be introduced at a different level.

Improved approach

Old:

- sentence $\rightsquigarrow$ character
- character: contexts $\rightarrow$ contents (resolution conditions)

New:

- sentence $\rightsquigarrow$ character (resolution conditions)
- character $\subseteq \{f \mid f\text{: contexts} \rightarrow \text{contents}\}$

This means that:

- An information state becomes a function from contexts to contents, rather than a set of worlds.
- Equivalently: a set of context-world pairs.

Semantics

An information state s is a set of context-world pairs.

We write $s \models_f \varphi$ to indicate that information state s supports φ under assignment function f.

- $s \models_f Qa_1...a_n \iff$ for all $\langle c, w\rangle \in s$: $\langle [a_1]_{cfw}, ..., [a_n]_{cfw}\rangle \in [Q]_{cfw}$
- $s \models_f a = b \iff$ for all $\langle c, w\rangle \in s : [a]_{cfw} = [b]_{cfw}$
- $s \models_f \bot \iff s = \emptyset$
- $s \models_f \varphi \wedge \psi \iff s \models_f \varphi$ and $s \models_f \psi$
- $s \models_f \varphi \bigvee \psi \iff s \models_f \varphi$ or $s \models_f \psi$
- $s \models_f \varphi \to \psi \iff$ for all $t \subseteq s : t \models_f \varphi$ implies $t \models_f \psi$
- $s \models_f \exists x\varphi \iff$ there is some $a \in A$ such that $s \models_{f^x_a} \varphi$

$\star$ $s \models_f A\varphi \iff \{\langle c, c\rangle \mid \langle c, w\rangle \in s\} \models_f \varphi$

Necessity and apriority

- φ expresses a necessary proposition in c iff the maximal information state in which c is fixed supports φ.

$$s \models_f \Box\varphi \iff \text{for all } \langle c, w\rangle \in s : \{c\} \times W \models_f \varphi$$

- φ is a priori iff the diagonal information state supports φ.

$$s \models_f \blacksquare\varphi \iff \{\langle c, c\rangle \mid c \in W\} \models_f \varphi$$

The diagonal information state encodes the information that the context and the world are the same.

Examples

Let @ be the actual world/context. Then:

- $\{\langle @, @\rangle\} \models \blacksquare ?\mathit{Located}(I, \mathit{here})$
  Because $\{\langle c, c\rangle \mid c \in W\} \models\ ?\mathit{Located}(I, \mathit{here})$
- $\{\langle @, @\rangle\} \not\models \square ?\mathit{Located}(I, \mathit{here})$
  Because $\{@\} \times W \not\models\ ?\mathit{Located}(I, \mathit{here})$
- $\{\langle @, @\rangle\} \not\models \blacksquare \exists x(x = I)$
  Because $\{\langle c, c\rangle \mid c \in W\} \not\models \exists x(x = I)$
- $\{\langle @, @\rangle\} \models \square \exists x(x = I)$
  Because $\{@\} \times W \models \exists x(x = I)$

Other frameworks

Other frameworks

Can we obtain similar results in other frameworks for question semantics?

Two alternative groups of approaches:

- Answer-set theories
- Partition theories

Answer-set theories

Answer-set theories analyze questions as sets of answers (Hamblin, 1973; Karttunen, 1977).

(12) Who came to the party?

This means (12) would be analyzed along these lines:
{ |John came to the party|, |Mary came to the party|, ...}

In this set-up, we could say that a question is a priori / necessary just in case its true answer is:

- Am I here now ⇒ I am here now
- Who am I? ⇒ I am Thom

Answer-set theories

However, answers are usually construed as propositions: at this level, we cannot distinguish apriority from necessity.

So answers would have to be construed as characters instead.

But the question 'Where are we?' is not a priori, even though it always has the true answer 'We are here'.

Answer-set theories

So there are some disadvantages of using a theory based on answer-sets:

- We need separate definitions for apriority and necessity for questions
- We have to supply a theory of answerhood that explains why 'We are here' is an answer to 'Are we here?', but not to 'Where are we?'

Partition theories

Groenendijk & Stokhof (1984): a question is an equivalence relation on the logical space, which holds between two worlds just in case the true complete answer to the question is the same in these worlds.

Polar questions:

$$w, v \models ?\alpha \iff w, v \models \alpha \text{ if and only if } w, w \models \alpha$$

What if we add this notion of questions to the standard two-dimensional framework?

For any question φ and any w:

$$w, w \models \varphi$$

So by the standard definition of apriority, all questions are a priori.

To check whether a question φ is a priori, we have to check whether its true answer is a priori. But we cannot recover this answer from the semantic value of φ:

	w	v
w	T	F
v	T	F

α

$\Downarrow$

	w	v
w	T	F
v	F	T

$?\alpha$

	w	v
w	T	F
v	F	T

β

$\Downarrow$

	w	v
w	T	F
v	F	T

$?\beta$

The problem is that in partition semantics, $?\alpha$ really expresses 'the actual true answer to the question whether α'. Unsurprisingly, this is a priori true.

So this theory has disadvantages as well:

- The standard definition of apriority does not extend to questions as they are represented in partition semantics.
- It is not clear how to extend the definition in such a way that it does apply.

Conclusion

Conclusion

- We combined insights from two-dimensional semantics and inquisitive semantics.
- This resulted in definitions of apriority and necessity in terms of information rather than truth.
- These definitions apply to questions as well as statements.
- The character of a statement or question should not be a function from contexts to sets of sets of worlds, but rather a set of sets of context-world pairs.

Bonus: Actuality operator

The addition of the *A* operator in inquisitive semantics allows us to give an analysis of sentences like (13):

(13) John does not know which (**actual**) students are students.

There is a reading of ‘which students are students?’ which is not trivial, but in which the first occurrence of students is anchored to the actual world.

Bonus: Actuality operator

The occurrence of a (silent) A in questions can possibly be an explanation for the perceived asymmetry of *which*-questions:

$$\text{Which Russians are spies?} \neq \text{Which spies are Russian?}$$

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